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SP

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MATHS (Higher)

EXAM BOARD: EDEXCEL **COURSE CODE: 1MA1/H**

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TOPIC AREA KEY

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Name: .... Tutor Group: ....

MATHS (Higher) SP - TOPIC 1
Bounds and Estimation

INTRODUCTION
Any recorded measurement has almost certainly been rounded. The true value will be somewhere between the ***lower bound*** and the ***upper bound***.

When a value is rounded you round to a ***degree of accuracy*** e.g. nearest 10, 2 decimals places, 1 significant figure, etc.

KEY WORDS

Rounding	Making a number 'simpler'. When a number is rounded it is less accurate.
Accuracy	Accuracy is how close a measured value is to the actual (true) value.
Degree of Accuracy	The degree to which a given value is correct e.g. to nearest cm, to 1 decimal place, etc.
Upper Bound	The largest number that rounds down to the given value.
Lower Bound	The smallest number that rounds up to the given value.
Estimation	To make an approximate calculation based on rounding.
Limits of Accuracy	The upper and lower bounds are sometimes known as the limits of accuracy.
Error Interval	The range between the limits of accuracy.

FURTHER LINKS
Corbett Maths –under *'Videos and Worksheets'* tabs:

Limits of Accuracy – Video 183 and 184, practice questions, 2 different textbook exercises.

HegartyMaths:
Clips and tasks: 137, 138, 139

JustMaths:
Google: STICKY! 9-1 Exam questions by topic – HIGHER TIER – version 2

EXAM TIPS:

Calculating Upper and Lower Bounds
You should use the "half a unit" rule to get your upper and lower bounds. The upper bound is "half a unit above" and the lower bound is "half a unit below".

EXAMPLE:
The length, L cm, of a line is measured as 13 cm correct to the nearest centimetre. State the upper and lower bound.

ANSWER:
1. Identify the degree of accuracy. In this case, the line was measured to the **nearest centimetre**.
2. Divide this by 2 → 1cm ÷ 2 = 0.5cm
3. Subtract this from our rounded value to calculate the lower bound → 13-0.5 = 12.5cm
4. Add the 0.5cm onto the rounded value to calculate the upper bound → 13 + 0.5 = 13.5cm

Stating the Error Interval
This must be written using a combination of inequality symbols and your upper and lower bounds.
You will always use the same 2 inequality symbols for every error interval.
It will always be formatted like this:

Lower Bound ≤ x < Upper Bound

EXAMPLE:
The length, L cm, of a line is measured as 13 cm correct to the nearest centimetre. State the error interval for L.

ANSWER:

12.5cm ≤ L < 13.5cm

We have stated the correct bounds, used the correct inequality symbols, and the question **told us to use 'L'** in our error interval.

Calculating with Bounds
You may be asked to calculate maximum or minimum values using bounds.
1. Determine what the questions is asking (maximum or minimum)
2. Identify what calculations you need to do
3. Select the correct bounds depending on what operation you are performing and whether you are looking for the maximum or the minimum value.

KEY FACTS TO MEMORISE

Operation	Minimum	Maximum
Addition $(a+b)$	$a_{min} + b_{min}$	$a_{max} + b_{max}$
Subtraction $(a-b)$	$a_{min} - b_{max}$	$a_{max} - b_{min}$
Multiplication $(a \times b)$	$a_{min} \times b_{min}$	$a_{max} \times b_{max}$
Division $(a \div b)$	$a_{min} \div b_{max}$	$a_{max} \div b_{min}$

EXAM QUESTIONS

1. The length of a line is 73 centimetres, correct to the nearest centimetre.
   (a) Write down the least possible length of the line.
   (b) Write down the greatest possible length of the line.

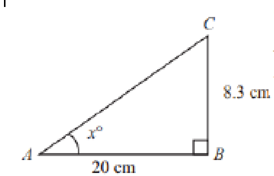
2. Sandeep takes 35 seconds, to the nearest second, to run a race.
Write down an error interval for the time. *t* seconds, taken to run the race.

3. A field is in the shape of a rectangle.
The length of the field is 340 m, to the nearest metre.
The width of the field is 117 m, to the nearest metre.
Work out the error interval for the perimeter, *p*, of the field.

4. $m = \dfrac{\sqrt{s}}{t}$

s = 3.47 correct to 2 decimal places. t = 8.132 correct to 3 decimal places. By considering bounds, work out the value of m to a suitable degree of accuracy.
You must show all your working and give a reason for your final answer.

STRETCH
Investigate problem solving with bounds in all contexts e.g.:



Triangle *ABC* is right-angled at *B*.
AB = 20 cm, correct to 1 significant figure.
BC = 8.3 cm, correct to 2 significant figures.

Calculate the lower bound for the value of tan x°

MATHS (Higher) SP – TOPIC 2

Fractional and Negative Indices

INTRODUCTION

Indices tell us how many times to use a number in a multiplication. You should already be familiar with the basic laws of indices:

$x^a \times x^b = x^{a+b}$

$x^a \div x^b = x^{a-b}$

$(x^a)^b = x^{ab}$

You will also need to answer questions where the index is negative or fractional (or both)

KEY WORDS

Base Number	The number that gets multiplied when using a power
Index	(Exponent/ Power) – The number of times the base number is used as a factor
Reciprocal	(also known as multiplicative inverse) You can find the reciprocal of a number (x) by calculating 1 divided by x $(\frac{1}{x})$
Irrational Number	Any number that cannot be written as a fraction where the numerator and denominator are integers
Unit Fraction	A fraction where the numerator is 1

FURTHER LINKS

Hegarty Maths Tasks 104 – 110

CorbettMaths video 173 and 175

EXAM TIPS:

If an expression contains a negative index it **does not** mean that the answer is negative

Ensure that if you write an index in an answer it is written clearly e.g. x^5 not $x5$

An easy way to find the reciprocal of a fraction is to flip the fraction e.g. $(\frac{4}{5})^{-1} = \frac{5}{4}$

KEY FACTS TO MEMORISE

$a^0 = 1$

$a^{-n} = \frac{1}{a^n}$

If the index is negative, take the positive index then find the reciprocal

e.g. $3^2 = 9$, so $3^{-2} = \frac{1}{9}$

$a^{\frac{1}{n}} = \sqrt[n]{a}$

A fractional index like $\frac{1}{n}$ means take the n-th root,

eg $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

Take the n[th] root (as above) then raise to the power of m

e.g. $4^{\frac{3}{2}} = (\sqrt{4})^3 = 2^3 = 8$

EXAM QUESTIONS

Find the value of the following:

a) 5^{-2}

b) $8^{\frac{1}{3}}$

c) $64^{\frac{2}{3}}$

d) $(\frac{9}{16})^{-\frac{1}{2}}$

Write 8 in the form 4^n

Write $\sqrt{125}$ in the form 5^n

Work out $27^{\frac{2}{3}} \div 9^{\frac{3}{2}}$

STRETCH

Put these numbers in ascending order

$8^{\frac{1}{2}}$ $4^{\frac{2}{3}}$ $32^{\frac{1}{3}}$ $2^{\frac{5}{6}}$

Find the value of x

$3^{2x+7} = 81$

MATHS (Higher) SP – TOPIC 3
Surds

INTRODUCTION

A surd is a square root for a number that is not a square number.

KEY WORDS

Rational	A number that can be expressed as a fraction
Irrational	A number that cannot be expressed as a fraction, e.g. π , $\sqrt{2}$
Rationalise	The process used to rewrite a fraction so that the denominator is a rational number
Expand	Multiply out the brackets
Square number	The product of a number multiplied by itself. E.g, 1, 4, 9, 16, 25, 36…
Denominator	The value on the bottom of a fraction

FURTHER LINKS

Hegarty Maths Clips:
- Multiplication and divison with surds – 113/114
- Simplifying surds – 115
- Expanding brackets with surds – 116/117
- Rationalising surds – 118/119

Corbet Maths:
- Videos 305 – 308

Pearson Textbook (Higher):
- Page 539-540

EXAM TIPS:

RULES OF SURDS

$$\sqrt{a} \times \sqrt{a} = a$$

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

SIPLIFYING SURDS

Find two factors – one should be the largest possible square number. Examples:

a) $\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

b) $\sqrt{8} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

c) $\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

ADDING AND SUBTRACTIONG

You can only add or subtract when the number in the root is the same

Examples:

a) $5\sqrt{3} + 6\sqrt{3} = 11\sqrt{3}$

b) $9\sqrt{2} + 5\sqrt{2} = 4\sqrt{2}$

EXPAND AND SIMPLIFY

$$(6 + \sqrt{5})(7 - \sqrt{5})$$

Multiply each of the terms:
- $6 \times 7 = 42$
- $6 \times -\sqrt{5} = -6\sqrt{5}$
- $\sqrt{5} \times 7 = 7\sqrt{5}$
- $\sqrt{5} \times -\sqrt{5} = -5$

$$42 - 6\sqrt{5} + 7\sqrt{5} - 5 = 37 + \sqrt{5}$$

RATIONALISING THE DENOMINATOR (BASIC)

Multiply the numerator and denominator by the surd. The denominator surds will cancel out.

$$\frac{42}{\sqrt{7}} = \frac{42}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{42\sqrt{7}}{7} = 6\sqrt{7}$$

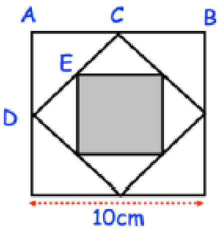
EXAM QUESTIONS

1. Evaluate $\sqrt{3} \times \sqrt{7}$
2. Evaluate $10\sqrt{8} \div 2\sqrt{2}$
3. Express $\sqrt{32}$ in its simplest form
4. Simplify $\sqrt{50} + \sqrt{32}$
5. Write $\sqrt{6} \times \sqrt{8}$ in the form $a\sqrt{3}$, where a is an integer
6. Rationalise the denominator of $\frac{12}{\sqrt{3}}$
7. Expand and simplify $(\sqrt{3} + \sqrt{5})^2$
8. Show that $(\sqrt{2} + 3\sqrt{8})^2 = 98$

STRETCH

The midpoints of the sides of a square of side 10cm are joined to form another square. This process is then repeated to create the shaded square.

Find the area of the shaded square



MATHS (Higher) SP – TOPIC 4

Recurring Decimals

INTRODUCTION

As well as converting between fractions and terminating decimals, you will be expected to do the same with recurring decimals (where one or more of the numbers repeat)

KEY WORDS

Terminating Decimal	The number stops after a certain number of decimal places
Recurring Decimal	Contains a pattern of numbers which repeat forever E.g. 0.7777… = $0.\dot{7}$ 0.803803803… = $0.\dot{8}0\dot{3}$
Rational	A number that can be expressed as a fraction
Irrational	A number that cannot be expressed as a fraction, e.g. π , $\sqrt{2}$

FURTHER LINKS

Hegartymaths (Clips 53 - 54)

Corbett Maths (Video clip 96)

EXAM TIPS:

To convert a fraction to a recurring decimal, use the bus stop method for division

$$\begin{array}{r} 0.2\ \ 4\ \ 2\ \ 4\ldots \\ 33\,\big)\,8.0\,{}^{14}0\,{}^{8}0\,{}^{14}0\,{}^{8}00 \end{array}$$

$$\frac{8}{33} = 0.\dot{2}\dot{4}$$

Converting a recurring decimal to a fraction:

- Let your recurring decimal be r
- Multiply r by a power of 10, so that the part that repeats moves to the left of the decimal
- You can now subtract to get rid of the decimal
- Divide to leave r (don't forget to simplify if possible)

Examples

Let $r = 0.\dot{2}3\dot{4}$

$1000r = 234.\dot{2}3\dot{4}$

$$\begin{array}{rl} 1000r &= 234.\dot{2}3\dot{4} \\ -\quad r &= \quad 0.\dot{2}3\dot{4} \\ \hline 999r &= 234 \end{array}$$

$$r = \frac{234}{999} = \frac{26}{111}$$

Let $r = 0.1\dot{6}$

$10r = 1.\dot{6}$

$100r = 16.\dot{6}$

$$\begin{array}{rl} 100r &= 16.\dot{6} \\ -\quad 10r &= \ \ 1.\dot{6} \\ \hline 90r &= 15 \end{array}$$

$$r = \frac{15}{90} = \frac{1}{6}$$

KEY FACTS TO MEMORISE

For a decimal to terminate, when written as a fraction its denominator will only have prime factors of 2 and/or 5

EXAM QUESTIONS

Prove that the recurring decimal $0.\dot{4}\dot{5} = \frac{15}{33}$

Express the recurring decimal $0.\dot{2}1\dot{3}$ as a fraction

x is an integer such that $1 \le x \le 9$. Prove that $0.\dot{0}\dot{x} = \frac{x}{99}$

STRETCH

Work out whether the following fractions are terminating decimals

$\frac{1}{28}$ $\quad\frac{13}{16}$ $\quad\frac{119}{125}$

Work out $0.\dot{7} \times 0.\dot{2}\dot{3}$

MATHS (Higher) SP – TOPIC 5

Linear Graphs and Equations of Straight Line

INTRODUCTION

As well as graphing straight lines you will be expected to find the equation of the line between two points, from the graph and given the y-intercept and the gradients.

KEY WORDS

Gradient	The steepness of the line (the change in y divided by the change in x)
Y-Intercept	The point where a line crosses the y-axis (x=0)
Rise	The vertical change between two points
Run	The horizontal change between two points
X-intercept	The point where the line crosses the x-axis (y=0)

FURTHER LINKS

Hegartymaths (Clips 205-213)

Corbett Maths (Video clip 186,187,192,193)

EXAM TIPS:

Clearly show the substitution steps.
Don't forget to draw a straight line through your points with the ruler.

Graphing a straight line

- Set x equal to any number and substitute into the equation
- Solve for y
- Write your answer in the form (x,y)
- Do this twice more
- Plot the points found and use your ruler to draw a straight line through your points and through the axes.

Alternative method

- $Y = mx + c$
- Plot the y-intercept.
- Use the gradient to find another point on the line
- Draw a straight line using a ruler through the points.

Examples

Graph the line $y = 3x + 2$

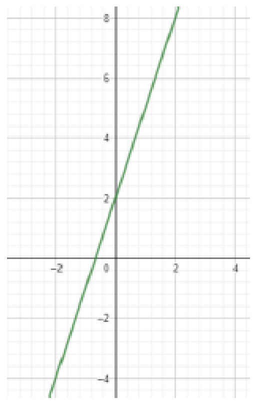
X	-2	0	1	2
Y	-4	2	5	8

X = -2
Y = 3x-2+2=-6+2=-4

X=0
Y= 3x0 + 2 = 2

X=1
Y=3x1+2=5

X=2
Y=3x2+2=8



KEY FACTS TO MEMORISE

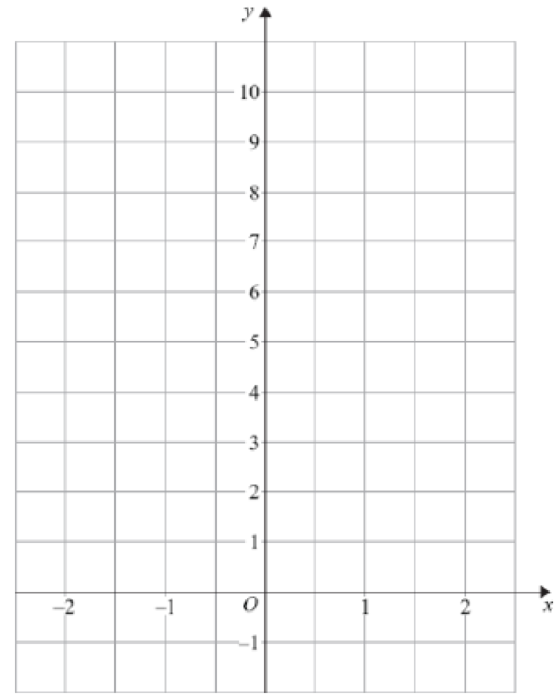
The gradient of a horizontal line is 0.
The gradient of a vertical line is undefined.
If the gradient is positive, the line rises.
If the gradient is negative, the line falls.

EXAM QUESTIONS

(a) Complete the table of values for $y = 2x + 5$

x	-2	-1	0	1	2
y	1		5		

(b) On the grid, draw the graph of $y = 2x + 5$ for values of x from $x = -2$ to $x = 2$



STRETCH

Draw the graph of $3x+5y=2$

Find a solution to $3x+8=2x+9$ graphically.

Graph $y = x^2+3x+2$

MATHS (Higher) SP – TOPIC 6

Quadratic, Cubic and Reciprocal Graphs

INTRODUCTION

These 3 graphs are curves.
You will need to be able to sketch each of them using a table of values and each one is generally worth a minimum of 4 marks in your GCSE exams. You will also need to be able to recognise each one and pair it up with the type of equation which generates it as some questions will ask you to do this.

KEY WORDS

Quadratic	An algebraic function where the highest index number is a 2.
Cubic	An algebraic function where the highest index number is a 3.
Reciprocal	An algebraic function that involves division by one of the variables, often shown as a fraction.

FURTHER LINKS

Hegartymaths (Clips 251, 298, 299, 300, 301)

Corbett Maths (Clips 367c, 367d, 371, 344, 346)

LINKED TOPICS

Can be part of a Quadratic simultaneous equation question.

Volumes can sometimes be written as Cubic equations.

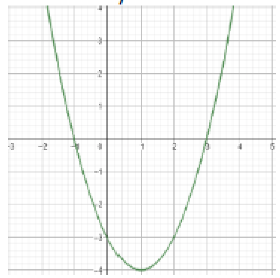
Circle graphs are also fairly common.

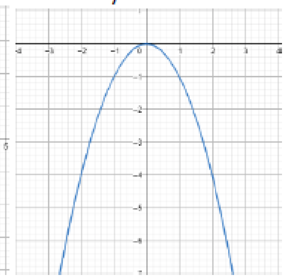
KEY FACTS TO MEMORISE

Recognising the functions/graphs

A quadratic graph is "u" shaped or possibly "n" shaped. The equation will have an x^2.

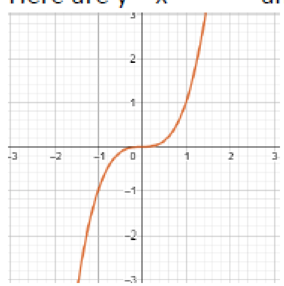
Here are $y = x^2-2x-3$ and $y = -x^2$

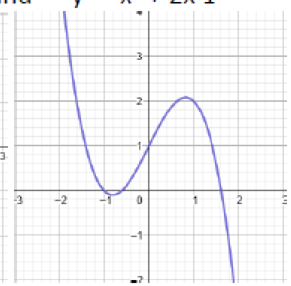




A cubic graph is "N" shaped or perhaps a stretched backwards "s". The equation will have an x^3.

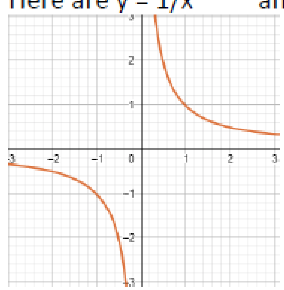
Here are $y = x^3$ and $y = -x^3 + 2x$ 1

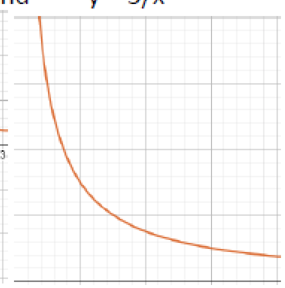




A Reciprocal graph can be in 2 parts or often just one "L" shaped part.

Here are $y = 1/x$ and $y = 3/x$





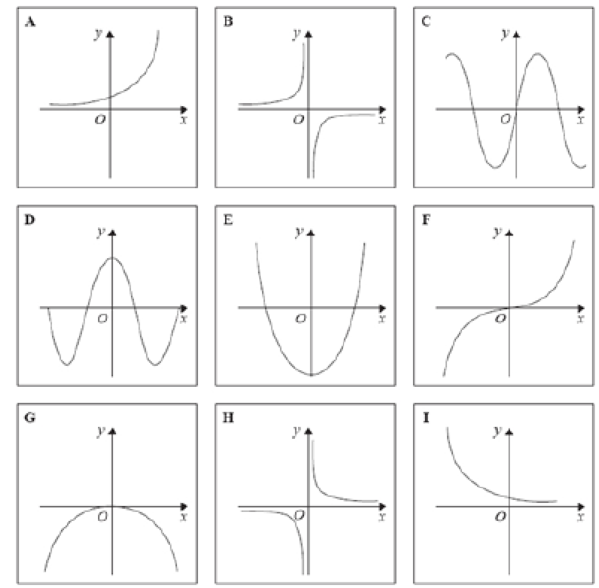
EXAM TIPS:

If your graph looks crazy it is quite possibly wrong. Re-check your table-of-values, especially where x is negative.

Plot all points carefully and join together with a smooth curve drawn with a pencil.

EXAM QUESTIONS

Here are some graphs.



In the table below, match each equation with the letter of its graph.

Equation	Graph
$y = \sin x$	
$y = x^3 + 4x$	
$y = 2^x$	
$y = \frac{4}{x}$	

MATHS (Higher) SP - TOPIC 7

Factorisation and the Quadratic Formula

INTRODUCTION

There are 3 algebraic ways to solve quadratic equations. You can also solve quadratic equations using a graph. Two of the algebraic ways to solve quadratic equations are Factorisation and the quadratic formula.

When we solve quadratic equations, there will be two solutions.

KEY WORDS

Equation	Mathematical statement that 2 expressions are equal
Quadratic	An equation which has the general form $ax^2 + bx + c = 0$
Factorise	Writing a number or another mathematical object as a product of several factors, usually smaller or simpler objects of the same kind
Formula	An equation that shows the relationship between different variables which is used to solve a problem
Substitute	Replacing a variable with a number
Coefficient	Constant which is multiplied by a variable. E.g. in $ax^2 + bx + c = 0$ the coefficient of x^2 is a.

FURTHER LINKS

Corbett Maths –under 'Videos and Worksheets' tab:

Factorisation (solving) – Video 226, practice questions, textbook exercise

Quadratic Formula – Video 227, practice questions, textbook exercise

HegartyMaths:

Clips and tasks: 230-233 (solving by factorising), 241-242 (solving using the quadratic formula)

JustMaths:

Google: STICKY! 9-1 Exam questions by topic – HIGHER TIER – version 2

EXAM TIPS:

1. To solve a quadratic it must equal 0. Sometimes this will mean you need to rearrange the equation.
2. If the paper is non-calculator the quadratic often factorises.
3. When using the quadratic formula, write out your values for a, b and c before attempting to substitute into the quadratic formula.
4. ALWAYS lay out your working neatly.
5. Show ALL your steps.
6. When you have factorised, expand your brackets to check your factorisation was correct.

KEY FACTS TO MEMORISE

Solving by factorising:

Once you have factorised, your brackets will be equal to 0.
As at least one of these brackets must equal 0. We then have 2 linear equations to solve.

EXAMPLE:

$(x + 3)(x - 5) = 0$

Either $x + 3 = 0$ or $x - 5 = 0$

Therefore $x = -3$ or $x = 5$

Solving using the quadratic formula:

You must memorise the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Which solves $ax^2 + bx + c = 0$

EXAM QUESTIONS

1. Solve $x^2 + 10x - 24 = 0$

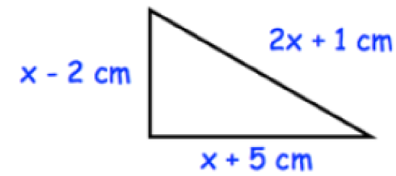
2. Solve. $m^2 + 5m = 24$

3. Solve $y^2 - 6y - 8 = 0$
Write your answer in the form $a \pm \sqrt{b}$ where a and b are integers.

4. Victor is y years old.
His brother Fred is four years old than Victor.
The product of their ages is 780.
(a) Set up an equation to represent this information.
(b) Solve your equation from (a) to find Victor's age

5. Solve the equation $4x^2 + x - 7 = 0$ Give your answers to two decimal places.

6.



Shown is a right angled triangle.

(a) Show that $x^2 - x - 14 = 0$

(b) Find x.

STRETCH

1. Sketch a quadratic curve (parabola) after having found its roots. Where is the turning point?

2. Why might this equation give you a maths error (on your calculator)?
$x^2 + 4x + 5 = 0$

Hint: what does it look like on a graph?

MATHS (Higher) SP - TOPIC 8

Simultaneous equations

INTRODUCTION

Two equations which cannot be solved on their own but must be solved together are referred to as simultaneous equations.
Often both equations have both x and y as unknowns and finding the correct pair of solutions is equivalent to finding the points where two lines cross.
Linear simultaneous equations will have one pair of solutions, whilst swapping one of the equations for a quadratic equation gives 2 pairs of solutions.
Questions can be asked in context, requiring you to write the equations yourself and re-interpret the answers afterwards.
There are two main methods used to solve these algebraically (without a graph), Elimination or Substitution.
We generally teach elimination for solving linear simultaneous equations and substitution for questions involving a quadratic equation.
Some questions may get you to solve the simultaneous equations graphically by drawing them.

KEY WORDS

Unknown	The letters in the equation. The values you are trying to work out.
Coefficient	The number just in front of an unknown. For 3x the coefficient would be 3.

KEY FACTS TO MEMORISE

EXAM TIPS:

You get no marks for numbering the equations but it really helps you to stay organised and present your work clearly to help the examiner to award you full marks.

Elimination Method

Try using the acronym **MESS** which stands for

Match **E**liminate **S**olve **S**ubstitute

It really helps to number your equations. Use numbers in circles on the left of your equations. You can then write down on the right hand side how you are making each equation.
Here is a worked example:

Solve the following simultaneous equations

$3x + 2y = 4$

$4x + 5y = 17$

Label the equations

1 $3x + 2y = 4$

2 $4x + 5y = 17$

Multiply up one or both equations so that the coefficients of either the x or the y **match**, do not worry about positive or negative.

3 $15x + 10y = 20$ 1 x5

4 $8x + 10y = 34$ 2 x2

Now add or subtract two equations to **eliminate** one of the unknowns.

5 $7x = -14$ 3 - 4

Now **solve** to find one unknown.

(÷7) $x = -2$ (÷7)

Substitute this value back into one of the original equations and solve that for the other unknown.

6 $3x(-2) + 2y = 4$ sub into 1

$-6 + 2y = 4$

$2y = 10$

$y = 5$

Solutions are $x = -2$, $y = 5$

Substitution Method

Solve these simultaneous equations.

$y = x^2 - 35$

$x - y = 5$

Rearrange the linear equation to get "x=" or "y="

X = <u>y + 5</u>

Substitute the underlined part into the quadratic equation in place of the X, then solve.

$Y = (y + 5)^2 - 35$

$Y = y^2 + 10y + 25 - 35$

$0 = y^2 + 9y - 10$

$0 = (y + 10)(y - 1)$

$Y = -10$ and $y = 1$

Substitute these 2 solutions into the linear equation to get the other 2 solutions.

$X - -10 = 5$ so $x = -5$ when $y = -10$ and

$X - 1 = 5$ so $x = 6$ when $y = 1$

EXAM QUESTIONS

1.

Solve

$$4x + 3y = 19$$
$$3x - 5y = 7$$

2.

Solve these simultaneous equations.

$$x^2 + y^2 = 26$$

$$y + 6 = x$$

LINKED TOPICS

Forming and solving equations.
Changing the subject of a formula.
Solving quadratic equations.

FURTHER LINKS

Hegartymaths (Clips 190 – 195, 218 – 219, 259, 246)
Corbett Maths (Clips 295 - 297)

MATHS (Higher) SP - TOPIC 9

Completing the Square

INTRODUCTION

You must know how to complete the square and rearrange a quadratic as well as find the turning points of a quadratic using this method.

KEY WORDS

Coefficient	Constant which is multiplied by a variable. E.g. in $ax^2+bx+c=0$ the coefficient of x^2 is a.
Factorise	Writing a number or another mathematical object as a product of several factors, usually smaller or simpler objects of the same kind
Perfect Square	An expression in the form of $(a+b)^2$ or $a^2+2ab+b^2$

FURTHER LINKS

Hegartymaths (Clips 235-239)

Corbett Maths (Video clip 10)

EXAM TIPS:

What do you need to add to the expression to turn it into $(a+b)^2$?

Completing the square for $y=ax^2+bx+c$

- Factor out a of the first two terms if a is not 1
- Half the coefficient of x and square your result. Add and subtract this to the RHS.
- Factorise the perfect square and simplify the remaining two terms.
- $Y=a(x-h)^2+k$

Examples

$Y = x^2+6x+4$
$= x^2+6x+(3)^2-(3)^2+4$
$= (x+3)^2-9+4$
$= (x+3)^2-5$

$Y = 2x^2-4x-7$
$= 2(x^2-2x)-7$
$= 2[x^2-2x+(-1)^2-(-1)^2]-7$
$= 2[(x-1)^2-1]-7$
$= 2(x-1)^2-2-7$
$= 2(x-1)^2-9$

KEY FACTS TO MEMORISE

What you add on to make the expression factorise to be $(a+b)^2$, you also have to take away.

EXAM QUESTIONS

(*a*) Express $x^2+4x-12$ in the form $(x+a)^2+b$

Write $2x^2+7x-3$ in the form $a(x+m)^2+n$.

STRETCH

Solve $2x^2-4x-7=0$ by complete the square

What is the minimum value of $y = x^2+6x+4$?

What is the maximum value of $y = -2x^2+6x+4$?

MATHS (Higher) SP - TOPIC 10

Function notation and Iteration

INTRODUCTION

A function is a relationship between two sets of values. We use function notation f(x). You say this as "f of x". x is the input value and f(x) is the output value.
Iteration is the act of repeating a process. Iteration is a way of solving equations. It is often used as a means of obtaining successively closer approximations to the solution of a problem. You would usually use iteration when you cannot solve the equation any other way.

KEY WORDS

Iteration	The act of repeating a process, often with the aim of approximating a desired result more closely.
Roots	Another word for solutions
Recursive Notation	For example: $x_{n+1} = \sqrt{3x_n + 6}$ x_n is the nth term. x_{n+1} is the term after the nth term.

FURTHER LINKS

Corbett Maths –under *'Videos and Worksheets'* tab:
Iteration – Video 373, practice questions

HegartyMaths:
Clips and tasks: 322

JustMaths:
Google: STICKY! 9-1 Exam questions by topic – HIGHER TIER – version 2

EXAM TIPS:

Exam questions are commonly in three parts and worth about 6 marks. Part a, would often ask you to show between which integers the roots lie. Part b, would often ask you to show how the equation could be rearranged Part c, would often ask you to carry out some iterations to find a solution.

To show an equation has a root between 2 values

1. ALWAYS ensure the equation is equal to 0
2. Use function notation
3. Input, in turn, the given values.
4. The outputs should differ in sign (positive/negative)
5. You MUST write statement similar to *"... as there is a change in sign there is at least one solution between [the given values]".*

Showing how the equation is rearranged

This rearrangement is different to usual as you are not creating an equation with only x as the subject, it will appear twice.

1. Show ALL your steps as this is the only place to gain marks in this question
2. Keep referring to the end goal.

Carry out iterations to find a solution

1. You will be given the first value to input into the iterative formula, usually x_0 or x_1
2. Generate the first iteration manually so that when you can check you have inputted into your calculator correctly.
3. Keep substituting in your previous answer until your answers are the same to a certain degree of accuracy OR until the question asks you to stop (e.g. find x_1, x_2, and x_3)

EXAM QUESTIONS

1. (a) Show that the equation $x^3 - 10x = 30$ has a solution between $x = 4$ and $x = 5$

(b) Show that the equation $x^3 - 10x = 30$ can be arranged to give $x = \sqrt[3]{30 + 10x}$

(c) Starting with $x_0 = 4.5$ use the iteration formula $x_{n+1} = \sqrt[3]{30 + 10x_n}$ to find an estimate for the solution of $x^3 - 10x = 30$ to 2 decimal places.

2. (a) Complete the table for $y = x^3 - 5x + 4$

x	0	1	2	3	4
y		-8			40

(b) Between which two consecutive integers is there a solution to the equation?
$x^3 - 5x + 4 = 0$? Give a reason for your answer.

(c) Show that the equation $x^3 - 5x + 4 = 0$ can be arranged to give $x = \sqrt[3]{5x + 4}$

(d) Starting with $x_0 = 2.5$ use the iteration formula $x_{n+1} = \sqrt[3]{5x_n + 4}$ to find an estimate for the solution of $x^3 - 5x + 4 = 0$ to 2 decimal places.

STRETCH

A sequence of numbers is formed by the iterative process
$a_{n=1} = (a_n)^2 - a_n$

a) Describe the sequence of numbers when a_1=1
Show working to justify your answer.

b) Describe the sequence of numbers when a_1= -1
Show working to justify your answer.

c) Work out the value of a_2 when a_1= 1 - $\sqrt{2}$

MATHS (Higher) SP - TOPIC 11

Non-Linear Sequences

INTRODUCTION

A sequence is a list of numbers that are in an order, typically making a pattern of some sort.

KEY WORDS

Term	A number in a sequence
Term to term rule	The rule to get from one term to the next
Nth term rule	The expression which can be used to generate any term in a sequence
Linear/Arithmetic Sequence	A sequence where the term to term rule is a constant addition or subtraction
Geometric Progression	A sequence where the term to term rule is a constant multiplication
Fibonacci Sequence	A sequence where each term is the sum of the two previous terms
Quadratic Sequence	A sequence where the nth term contains an n^2

FURTHER LINKS

Hegarty maths
197-198 Linear Sequences
249 – Quadratic Sequences
263 – Fibonacci Sequences
264 – Geometric Sequences

EXAM TIPS:

<u>Finding the nth term of a quadratic sequence</u>
Example: 6, 11, 18, 27, 38
Start by finding the "second difference".

$$\begin{array}{ccccccccc} 6 & & 11 & & 18 & & 27 & & 38 \\ & 5 & & 7 & & 9 & & 11 & \\ & & 2 & & 2 & & 2 & & \end{array}$$

The second difference should be a constant. In this case it is 2. Halve the 2^{nd} difference to find the n^2 co-efficient. For this question, there will be one n^2.

(Note: If you are generating $2n^2$ or any other multiple of n^2, square first and then multiply by the co-efficient)

Generate the first 5 terms of n^2. (Substitute in the numbers 1 to 5.)

n^2 : 1 4 9 16 25

Subtract these numbers from the original sequence. This will create a linear sequence.

$$\begin{array}{cccccc} & 6 & 11 & 18 & 27 & 38 \\ - & 1 & 4 & 9 & 16 & 25 \\ \hline & 5 & 7 & 9 & 11 & 13 \end{array}$$

Find the nth term of the linear sequence.
It increases by 2 from each term to the next (2n).
To get from 2 to the first term, 5, add 3 (2n + 3).

Combine the linear sequence with your n^2 term to make the nth term of the quadratic sequence.

$$n^2 + 2n + 3$$

KEY FACTS TO MEMORISE

The most common Fibonacci sequence goes
1, 1, 2, 3, 5, 8, 13, 21, 34, ...

EXAM QUESTIONS

For each of the following sequences
a) Find the next two terms
b) Find the nth term rule
c) Find the 50^{th} term

2, 8, 16, 26, 38, ...

7, 15, 27, 43, 63, ...

1, 7, 19, 37, 61, ...

7, 5, 1, -5, -13, ...

The first 3 terms of a Fibonacci sequence are a, b and a + b. Show that the 6^{th} term of this sequence is 3a + 5b.

Here is a sequence.
2, $2\sqrt{7}$, 14, $14\sqrt{7}$, ...
a) Find the next two terms
b) Find the nth term rule

STRETCH

A company invests £65,000 into a savings account in the year 2014. Through interest, the investment grows by 12% each year from the start of the year to the end.
a) What this the common ratio for the geometric sequence?
b) Write the nth term of the geometric sequence.
c) How much profit will have been made at the end of 2020?

MATHS (Higher) SP - TOPIC 12

Tree Diagrams

INTRODUCTION

Tree diagrams allow us to see all the possible outcomes of an event and calculate their probability. Each branch in a tree diagram represents a possible outcome.

KEY WORDS

Probability	The likelihood of something occurring.
Event	A set of outcomes of an experiment i.e. the event of rolling a 2 when rolling a die.
Dependent	Two events are dependent if the probability of one happening changes the probability of the other happening
Independent	Two events are independent when the outcome of one does not affect the outcome of the other.
Mutually exclusive events	Events that cannot occur at the same time. E.g. You cannot get a 4 and an odd number on a dice at the same time
Conditional	Conditional probability is the probability that an event will happen given that another event has happened.

FURTHER LINKS

Hegarty maths clips: 354, 384, 386, 361-363.

Corbettmaths videos: 252

EXAM TIPS:

-When reading the question, first determine whether the events are dependent on each other or not.
Check whether or not the question is with replacement

Probabilities of all possible events add up to 1.
-Express probabilities only as fractions, decimals or percentages

Probabilities will always be between 0 and 1 (or 100%)

KEY FACTS TO MEMORISE

- The sum of the probabilities of all possible events equals 1.
- We multiply the probabilities of the events from the probability tree (see first example).
- We add independent probabilities i.e. probability of rolling a 2 or a 3 on a regular die is $\frac{1}{6}+\frac{1}{6}=\frac{2}{6}$

EXAM QUESTIONS

Question 1: There are 10 counters in a bag, 7 are blue and 3 are red. One counter is picked and replaced, then a second counter is picked. Show this in a probability tree diagram.

Question 2: A bag contains 4 red marbles and 5 green marbles. We draw a marble out of the bag. We then draw another marble. What is the probability the two marbles we took are both red?

Question 3: There are green and blue counters in a container.
Kevin takes at random a counter from the container.
Kevin takes at random a second counter from the container.
(a) Draw a tree diagram to show this.

(b) Work out the probability that Kevin picks counters that are different colours.

MATHS (Higher) SP - TOPIC 13

Venn Diagrams & Two-Way Tables

INTRODUCTION

Venn Diagrams and Two-Way tables are used to sort numbers, items or amounts into different combinations of categories.
They often involve questions on probabilities.

KEY WORDS

Sets	A group of items or numbers, typically all meeting a common rule.
Element	Each item or number in a set is called an element.
Intersect	Where elements or numbers belong to two different sets in a Venn Diagram
Union	Where elements or numbers belong to either one of two different sets or both in a Venn Diagram.
Compliment	The opposite of a set. A set and its compliment make up the universal set.
Universal set	The set of all elements in a Venn Diagram.

FURTHER LINKS

Hegarty maths
Venn Diagrams – 377-391
Two-Way Tables – 422-424

EXAM TIPS:

Two-Way Tables
In a two-way table, all of the rows and all of the columns should add up to a sum at the end. In the bottom right corner you should have the total amount.
e.g.

	French	German	Spanish	Total
Boys	27	15	11	53
Girls	15	19	13	47
Total	42	34	24	100

When asked probability questions, read the question very carefully. Give your probability as a fraction unless asked to do otherwise.

If choosing somebody at random, what is the probability that they do French? $^{42}/_{100}$

If choosing a **boy** at random, what is the probability that they do French? $^{27}/_{53}$

Venn Diagrams
A Venn diagram may contain a group of items (usually numbers) or just a frequency.

When drawing a Venn diagram, remember to draw a box around the outside for anything that is part of the universal set, but doesn't belong in the sub-sets.

Again, remember to read any probability questions very carefully.

When creating or completing a Venn diagram, it is usually easiest to start from the middle and work outwards to take any intersections into account first.

KEY FACTS TO MEMORISE

The intersection of two sets on a Venn diagram is the overlap.
A∩B means "in A and in B".

The union of two sets on a Venn diagram is everything in each set, including what is in both.
A∪B means "in A or in B or in both".

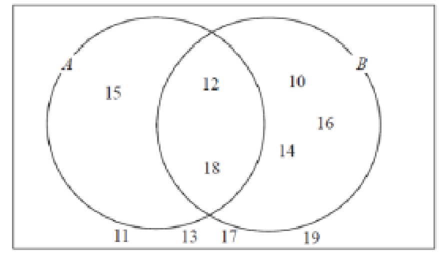
The compliment of a set means anything that is not in that set.
A' means "not in A".

EXAM QUESTIONS

Sami asked 50 people which drinks they liked from tea, coffee and milk.

48 people like at least one of the drinks.
19 people like all three drinks.
16 people like tea and coffee but do not like milk.
21 people like coffee and milk.
24 people like tea and milk.
40 people like coffee.
1 person only likes milk.

Create a Venn diagram for this information.

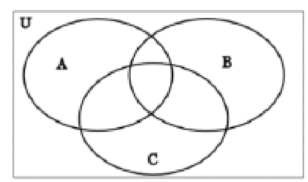


Write down the numbers in the set...
a) A∩B
b) A∪B
c) A'

	Walk	Bike	Car	Total
Boys		17		
Girls		13	17	52
Total	39			100

If somebody is chosen at random, what is the probability that they cycle to school?
If a girl is chosen at random, what is the probability that they walk to school?

STRETCH



Shade in the following sets.

(A∪B)∩C'
(A∩B)'∪C
A∪(B∩C)

MATHS (Higher) SP - TOPIC 14

Cumulative frequency (CF) and box plots

INTRODUCTION

CF graphs help us to estimate values from grouped frequency tables
Box plots help interpret and compare data using simple diagrams

KEY WORDS

Cumulative	The running total
Median	The value approximately half way through a set of numerical data in ascending order
Upper quartile (UQ)	The value approximately ¾ of the way through a set of numerical data in ascending order
Lower quartile (LQ)	The value approximately ¼ of the way through a set of numerical data in ascending order
Interquartile range (IQR)	A type of range found by subtracting the LQ from the UQ (this is a measure of how spread out the middle 50% of the data is)
Outlier	Individual pieces of data that lie outside of the pattern of data.
Box plot	A way of displaying data to show the median and quartiles (aka box and whisker diagram)

FURTHER LINKS

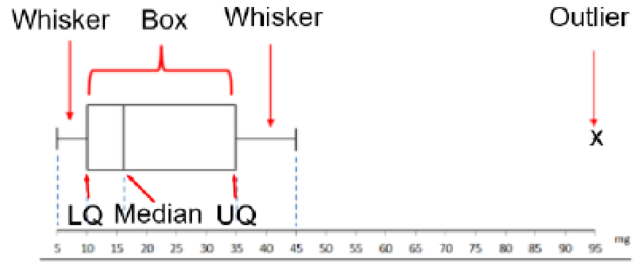
Hegarty Maths clips: 437, 438, 439, 440

Edexcel GCSE (9 – 1) Higher text book (chapter 144)

EXAM TIPS:

BOX PLOTS

On a box plot outliers are marked with a cross (x)
The whiskers of a box plot are drawn to reach the first numbers not considered outliers.



CF GRAPHS

1) ALWAYS plot the CF against the higher value of the class. E.g. if the class is $25 \leq x < 35$ and the CF for this class is *12* then you plot the point *(35, 12)* on your CF graph.

2) ALWAYS draw a smooth curve through your points UNLESS otherwise asked.

3) To *estimate* the median, draw a line to the curve from the CF value half the way up and then down to the x-axis

4) To *estimate* the LQ, draw a line to the curve from the CF value a quarter of the way up and then down to the x-axis.

5) To *estimate* the UQ, draw a line to the curve from the CF value three-quarters of the way up and then down to the x-axis

COMPARING DISTRIBUTIONS

If comparing CF graphs or box plots you must compare two things: a) The IQR b) The medians

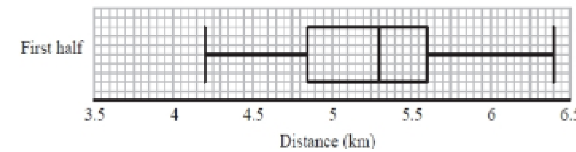
Anything else will score zero.

KEY FACTS TO MEMORISE

1) On a CF graph data values are plotted on the x-axis and cumulative frequencies along the y-axis
2) IQR – UQ – LQ
3) If asked for a suitable graph to find IQR and/or the median then a CF graph is good for grouped data. A box plot is good for raw data

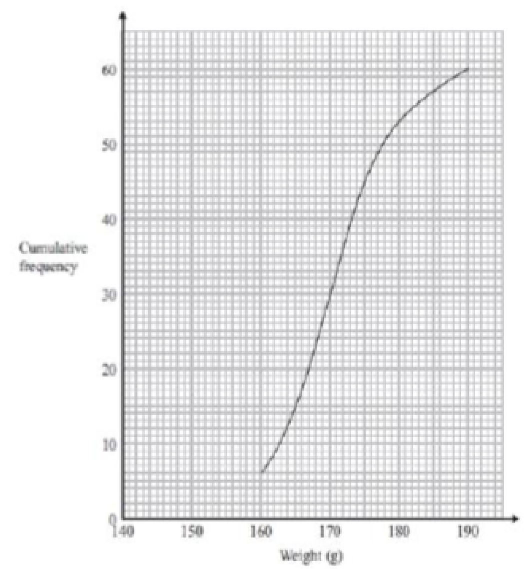
EXAM QUESTIONS

Colin took a sample of 80 football players.
He recorded the total distance, in kilometres, each player ran in the first half of their matches on Saturday.
Colin drew this box plot for his results.



(a) Work out the interquartile range.

Harry grows tomatoes.
This year he put his tomato plants into two groups, group A and group B.
Harry gave fertiliser to the tomato plants in group A.
He did not give fertiliser to the tomato plants in group B.
Harry weighed 60 tomatoes from group A.
The cumulative frequency graph shows some information about these weights.



(a) Use the graph to find an estimate for the median weight.

STRETCH

Interpret and compare box plots and CF graphs.

MATHS (Higher) SP - TOPIC 15

Histograms

INTRODUCTION

Histograms are used for continuous data, unlike bar charts which are used for qualitative data or discrete quantitative data.

KEY WORDS

Frequency density	Is found by dividing the frequency by the group. width
Group width/class interval	The difference between the highest possible value and the lowest possible value
Mean	The mean average found by adding all the pieces of data and dividing by how many there are.
Continuous	Continuous data is data that can be measured e.g. height, length, time etc.
Discrete	Discrete data is data that can be counted.
Quantitative	Quantitative data is data that can be measured or counted e.g. shoe size, lengths etc.
Qualitative	Qualitative data does not involve measurement or numbers e.g. colour.

FURTHER LINKS

Hegarty maths: 442-449

Corbettmaths videos: 157-159

KEY FACTS TO MEMORISE

Plotting a histogram

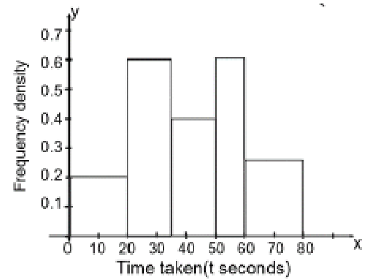
Frequency density – y-axis

To calculate this, divide the frequency of a group by the width of it.

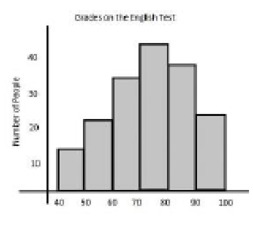
Time taken (t seconds)	Frequency	Frequency÷ group width	Frequency density
0<t≤20	4	4÷(20-0)	0.2
20<t≤35	9	9÷(35-20)	0.6
35<t≤50	6	6÷(50-35)	0.4
50<t≤60	6	6÷(60-50)	0.6
60<t≤80	5	5÷(80-60)	0.25

Group width – x-axis

To plot the histogram plot the group width(time taken) on the x-axis and the frequency density on the y-axis.



Estimating the mean from a histogram



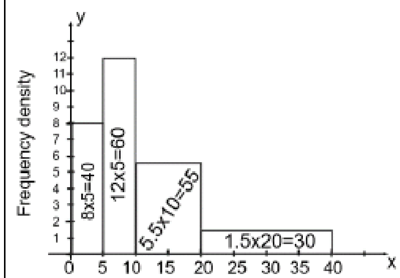
To estimate the mean multiply the frequency f each bar by the midpoint of its group interval. Add these up and divide by the total frequency.

Estimating the median

To find the place of the median divide the total frequency by 2. Then you can find the class interval containing the median by finding the cumulative frequency.

Reading from a histogram

The area of each bar represents the frequency of that group. To find the frequency we multiply the width(or class interval) with the height(frequency density).

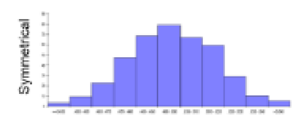


Frequency
40
60
55
30

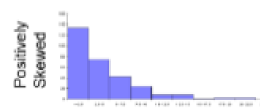
STRETCH

Interpreting distributions from histograms

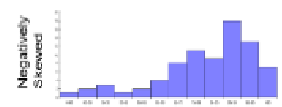
The shape of a histogram can tell us some keys points about the distribution of the data used to create it. It can tell us the relationship between the mean and the median, and allow us to describe the dispersion of the data.



The mean and median are roughly the same and are approximately in the centre. Data is evenly dispersed either side of the median



This suggests the mean is greater than the median. More of the data is towards the left-hand side of the distribution, with a few large values to the right.



This suggests the mean is less than the median. More of the data is towards the right-hand side of the distribution, with a few small values to the left.

MATHS (Higher) SP - TOPIC 16

Sampling, capture and re-capture

INTRODUCTION

Data is important as it helps us to understand things about our world and to make predictions (e.g. climate change or population growth)

KEY WORDS

Population	The set or group of people or things that you are interested in
Sample	A smaller number of people or things chosen from the population
Biased sample	A sample which does not represent the population fairly
Unbiased	A sample which does represent the population fairly
Census	A survey using the whole population
Strata	The name given to groups in a population e.g. male, female, green eyes, under 25 etc.
Stratified sample	A sample that contains groups in proportion to the groups in the population.

FURTHER LINKS

Edexcel GCSE Mathematics (Higher student book) Chapter 14

EXAM TIPS:

RANDOM SAMPLES

To be a genuinely random sample each item in the population must have an equal chance of being chosen.

CHOOSING RANDOM SAMPLES

To select a random sample you could use one of the following methods:

1) Draw names from a hat.
2) Generate random numbers on a calculator.
3) Use a table of random numbers.
4)

STRATIFIED SAMPLES

Use decimal multipliers to find the number of each item required.
e.g. if you require a sample of 24 items out of a population of 300 then the decimal multiplier is 0.08 (since $24 \div 300 = 0.08$)
If a sports club had 200 girls and 100 boys then the number of girls in the sample would be 16 ($0.08 \times 200 = 16$)

THE CAPTURE-RECAPTURE METHOD

Capture and mark a sample, say of size n

Release and recapture a sample of size M

Count the number of marked ones, say m

An estimate of the population is

$$\frac{n \times M}{m}$$

KEY FACTS TO MEMORISE

See definitions of key words

EXAM QUESTIONS

1) Explain why the following sample is biased: Ask 50 people at a bottle bank what they think of recycling.

2) A sports club manager wants to find out what members think of the facilities. There are 350 women and 450 men in the club.
   a) Explain why a stratified sample should be used.
   b) The club decides to ask 48 members. How many of each gender should be Asked

3) A scientist wishes to find out how many fish are in a lake.
   40 fish are caught and marked.
   Two weeks later, 40 more fish are captured and 5 of them have the mark.

   Estimate the size of the fish population.

STRETCH

Find out how to use random number tables to select samples.

Find out how to use your calculator to generate random numbers

Describe how you could select a random sample of 15 people from a population of 90.

MATHS (Higher) SP - TOPIC 17

Angles in parallel lines and polygons

INTRODUCTION

The words 'polygon' and 'parallel' comes from the Greeks. 'Poly' means many and 'gon' means angles. Parallel originates from the Greek word 'parállēlos' which means side by side.

KEY WORDS

Polygon	A 2D shape with straight sides that join together
Parallel	Two lines are parallel if the distance between them remains the same
Interior angles	Angles inside a polygon
Exterior angles	The angle between any side of a polygon and a line extended from the next side
Pentagon	A polygon with 5 sides
Hexagon	A polygon with 6 sides
Heptagon	A polygon with 7 sides
Octagon	A polygon with 8 sides
Nonagon	A polygon with 9 sides
Decagon	A polygon with 10 sides
Transversal	A line that passes through a pair of parallel lines

FURTHER LINKS

Hegarty Maths Clips:

Angles in polygons – 561/562/563/564

Angles in polygons with algebra – 565

Angles in parallel lines – 480/481/482/483

EXAM TIPS:

ANGLES IN POLYGONS

Sum of interior angles
$= (n - 2) \times 180$

n represents the number of sides the polygon has

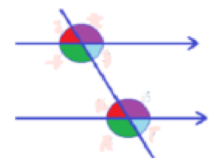
One interior angle of a regular polygon
$= \textit{Sum of interior angles} \div n$

Sum of exterior angles
$= 360°$

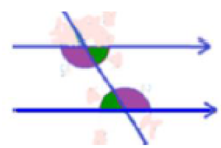
One exterior angle of a regular polygon
$= 360 \div n$

ANGLES IN PARALLEL LINES

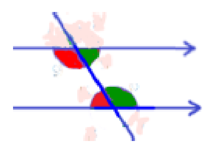
Corresponding Angles:
These are a pair of angles in matching corners and are equal.



Alternate Angles:
A pair of angles between the parallel lines but on opposite sides of the transversal



Co-interior Angles:
A pair of angles between the parallel lines that are on the same side of the transversal



KEY FACTS TO MEMORISE

- Angles around a point add up to 360°
- Angles on a straight line add up to 180°
- Vertically opposite angles are equal
- Angles in a triangle add up to 180°
- Angles in a quadrilateral add up to 360°
- Base angles of an isosceles triangle are equal
- All angles in an equilateral triangle are 60°

EXAM QUESTIONS

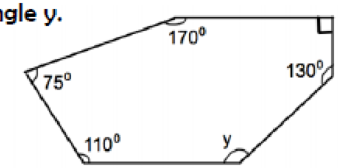
ANGLES IN POLYGONS

1. Work out the size of each interior angle of a regular pentagon

2. A regular polygon has 24 sides. Work out the size of each exterior angle

3. Each interior angle of a regular polygon is 124°.
Work out the number of sides

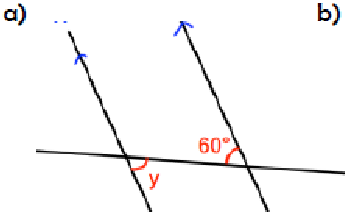
4. Calculate the size of angle y.



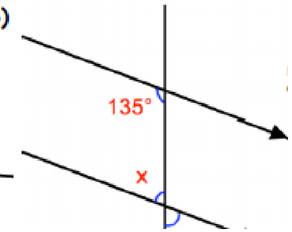
ANLGES IN PARALLEL LINES

1. In each question work out the size of the angles **and give reasons why.**

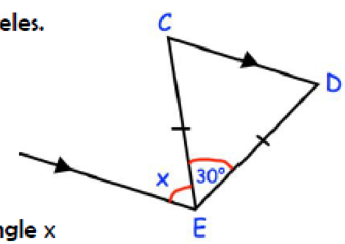
a)



b)



2. Triangle CDE is isosceles.
CD is parallel to FE.
Angle CED = 30°



Work out the size of angle x

STRETCH

Research and complete questions on angles in parallel lines and angles in polygons involving algebra and forming equations.

MATHS (Higher) SP - TOPIC 18

Trigonometry (right-angled) and special triangles

INTRODUCTION

Trigonometry is concerned with the calculation of the length of sides and angles in triangles.
Right-angled trigonometry is used with right-angled triangles.

KEY WORDS

Hypotenuse	The longest side of a right-angled triangle, opposite the right angle
Adjacent	Next to. In a right-angled triangle this is the side opposite the angle we are working with
Opposite	In a right-angled triangle this is the side opposite the angle we are working with *(the side that is not the adjacent or the hypotenuse!)*
Trigonometric ratio	The ratio of 2 sides and a related angle. Used t calculate unknown lengths or angles in right-angled triangles.
Sine (sin)	The trigonometric function that is equal to the ratio of the side opposite a given angle (in a right-angled triangle) to the hypotenuse
Cosine (cos)	The trigonometric function that is equal to the ratio of the side adjacent a given angle (in a right-angled triangle) to the hypotenuse
Tangent (tan)	The trigonometric function that is equal to the ratio of the sides opposite and adjacent to the given angle in a right-angled triangle

FURTHER LINKS

Corbett Maths –under *'Videos and Worksheets'* tabs

Trigonometry – Videos 329, 330, 331, practice questions, 3 textbook exercises

HegartyMaths:
Clips and tasks: 508 - 515

JustMaths:
Google: STICKY! 9-1 Exam questions by topic – HIGHER TIER – version 2

EXAM TIPS:

1. ALWAYS label the sides of your triangle as hypotenuse, adjacent or opposite first
2. ALWAYS Write out **S^{O}HC^{A}HT^{O}A**
3. Write out the trig ratio before substituting.
4. Remember when finding an angle you need to use the inverse function
5. Write out all the digits on your calculator before doing any rounding

KEY FACTS TO MEMORISE

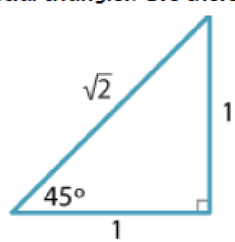
There are 3 trigonometric ratios to memorise:

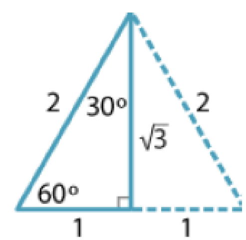
$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

Use the mnemonic **S^{O}HC^{A}HT^{O}A** to help you remember the 3 trig ratios.
Special triangles: Use these to memorise certain exact values

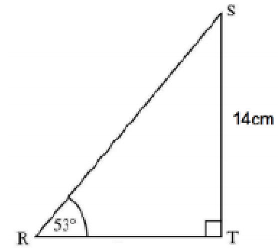




θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefined

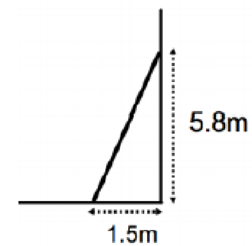
EXAM QUESTIONS

1. Calculate the length of side RT



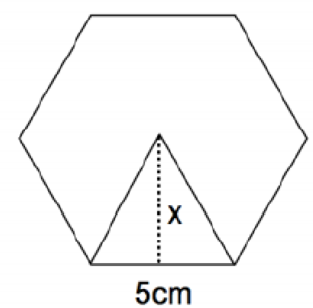
2.

A ladder is placed against a wall.
To be safe, it must be inclined at between 70° and 80° to the ground.



TRETCH

A regular hexagon can be divided into 6 equilateral triangles.
The diagram below shows one of the equilateral triangles.



(a) Calculate the height, x, of the equilateral triangle above.

MATHS (Higher) SP - TOPIC 19

Trigonometry (Non Right-Angled)

INTRODUCTION

You are to find the size of angles and the lengths between points in non-right angled triangles. You will be expected to apply this to finding areas and orienteering skills (bearings).

KEY WORDS

Side angle pair	The pair consists of a side and the corresponding opposite angle
Opposite side/a	The side or angle directly opposite the angle or side respectively.

FURTHER LINKS

Hegartymaths (Clips 521-524, 527-530,532-533)

Corbett Maths (Video clip 333-337)

EXAM TIPS:

Write out whether you are going to use Sine Rule or Cosine Rule.
Clearly show the formula that you will be substituting the values into.

KEY FACTS TO MEMORISE

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c} \text{ or } \frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

$$a^2 = b^2 + c^2 - 2 \times b \times c \times cosA$$

Don't forget BIDMAS for Cosine Rule

When to use the Sine Rule

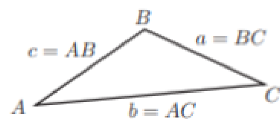
- If you have a side angle pair. (ASA or SSA or AAS)
- Make sure that the formula adaptation you are using places the unknown in the numerator.

When to use the Cosine Rule

- When you have all 3 sides of the triangle and want to find an angle. (SSS or SAS)
- Given two sides and the angle in between

Examples

$AB = 42$cm, $BC = 37$cm and $AC = 26$cm. Solve this triangle.



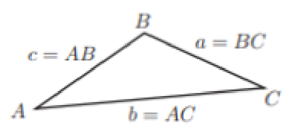
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$= 26^2 + 42^2 - 2(26)(42)\cos A$$

$$\frac{26^2 + 42^2 - 37^2}{(2)(26)(42)} = \frac{1071}{2184} = 0.4904$$

$$A = \cos^{-1} 0.4904 = 60.63°$$

$B = 21°$, $C = 46°$ and $AB = 9$cm. Solve this triangle.



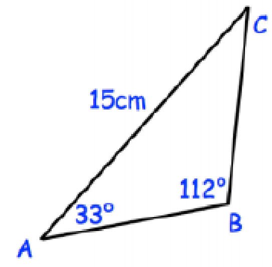
$$\frac{a}{\sin 113°} = \frac{b}{\sin 21°} = \frac{9}{\sin 46°}$$

$$\frac{b}{\sin 21°} = \frac{9}{\sin 46°}$$

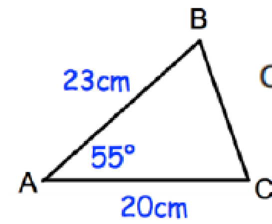
$$b = \sin 21° \times \frac{9}{\sin 46°} = 4.484\text{cm.} \quad (3dp)$$

$$a = \sin 113° \times \frac{9}{\sin 46°} = 11.517\text{cm.} \quad (3dp)$$

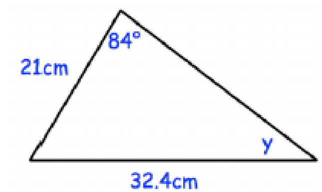
Exam Question



Work out the length of BC.

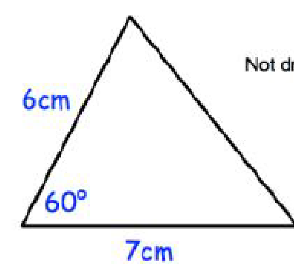


Calculate the length of BC.



Calculate the size of the angle labelled y.

STRETCH



Not drawn is scale.

Calculate the area of the triangle.

MATHS (Higher) SP - TOPIC 20

Compound Measures

INTRODUCTION

Compound Measures are combinations of two other measures. Speed is made up of distance and time. Formulae connect the measures together.

KEY WORDS

Speed	How far something travels in a given time.
Velocity	Speed in a given direction. Can be negative if travelling in the opposite direction.
Acceleration	The change in speed.
Density	How heavy something is (Mass) for a given size (Volume).
Pressure	How much force is exerted over a given area.

FURTHER LINKS

Hegarty maths
Speed – 716-724
Density – 725-733
Pressure – 734-737
Other compound units - 738

EXAM TIPS:

It is essential that you use the correct units in your working and in your answers. These can be obtained by reading the questions carefully.

e.g. If you are given a distance in km and a time in hours, the units for speed will be km/h

If you are given a mass in g and a volume in cm^3 the units for density will be g/cm^3.

You can also use this to help you remember a formula. If you are given a density in g/ml, this is a mass divided by a volume, hence density = mass ÷ volume.

You can also obtain formulae for other problems in this way. If a question states that water is coming out of a pipe at a rate of 60ml per second, then the formula is water flow = volume ÷ time.

In the higher paper, you are very likely to need to convert units. Make sure you are fluent at converting between units of time, distance and mass.

Always remember to consider if your answers and any part of your working is sensible for the problem you are solving.

KEY FACTS TO MEMORISE

Speed = Distance ÷ Time

Density = Mass ÷ Volume

Pressure = Force ÷ Area

EXAM QUESTIONS

Gary drove from London to Sheffield.
It took him 3 hours at an average speed of 80km/h.
Lyn drove from London to Sheffield.
She took 5 hours.
Assuming that Lyn drove along the same roads as Gary and did not take a break…

a) Work out Lyn's average speed from London to Sheffield.
b) If Lyn did not drive along the same roads as Gary, explain how this could affect your answer to part a.

A sculptor needs to lift a piece of marble.
It is a cuboid with dimensions 1m by 0.5m by 0.2m.
Marble has a density of $2.7g/cm^3$.
The sculptors lifting gear can lift a maximum load of 300kg.
Can the lifting gear be used to lift the marble?

A box exerts a force of 140N on a table.
The pressure on the table is $35N/m^2$.
Calculate the area of the box that is in contact with the table.

STRETCH

180g of copper is mixed with 105g of zinc to make an alloy.
The density of copper is $9g/cm^3$.
The density of zinc is $7g/cm^3$.

What is the density of the alloy?

MATHS (Higher) SP - TOPIC 21

Vectors

INTRODUCTION

You will be expected to work with vectors using 3 operations (+,-,x). You will be expected to find the magnitude and direction of vectors. You will be expected to solve geometric problems using vectors with links to ratio and fractions as well as properties of shapes.

KEY WORDS

Scalar	A quantity that only has magnitude
Vector	A quantity that has both direction and magnitude
Column vector	A vector in the form $\binom{a}{b}$
Magnitude	It is the length of the vector. The magnitude of the vector is denoted as $\lvert a \rvert$
Opposite Vector	The vector in the opposite direction $\binom{-a}{-b}$ if the given vector is $\binom{a}{b}$
Parallel Vectors	Vectors that are multiples of each other. Same direction, different magnitudes.

FURTHER LINKS

Hegartymaths (Clips 622-636)

Corbett Maths (Video clip 353,353a)

KEY FACTS TO MEMORISE

$$\binom{a}{b}$$

a: movement along the x-axis (left or right)
b: movement along the y-axis (up or down)

$-a$: movement left
$-b$: movement down

Examples

Operations with vectors

$$\binom{2}{6} + \binom{7}{-3} = \binom{9}{3}$$

If $b = \binom{4}{-2}$, then $3b = \binom{12}{-6}$

Magnitude of a vector

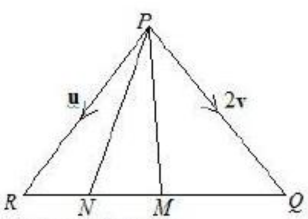
Calculate the magnitude of the vector $p = \binom{3}{7}$

$$|p| = \sqrt{3^2 + 7^2}$$

$$|p| = \sqrt{9 + 49}$$

$$|p| = \sqrt{58}$$

Geometric Problems with Vectors



Find in terms of u and v

a) $\overline{RQ}$ b) $\overline{RN}$ c) $\overline{PN}$

a) $\overline{RQ} = \overline{RP} + \overline{PQ}$ triangle law of vector addition

$= -\overline{PR} + \overline{RQ}$ negative vector

$= -\mathbf{u} + 2\mathbf{v}$

$= 2\mathbf{v} - \mathbf{u}$

b) $\overline{RN} = \frac{1}{4}\overline{RQ} = \frac{1}{4}(2\mathbf{v} - \mathbf{u}) = \frac{1}{2}\mathbf{v} - \frac{1}{4}\mathbf{u}$

c) $\overline{PN} = \overline{PR} + \overline{RN} = \mathbf{u} + \frac{1}{2}\mathbf{v} - \frac{1}{4}\mathbf{u} = \frac{3}{4}\mathbf{u} + \frac{1}{2}\mathbf{v}$

EXAM QUESTIONS

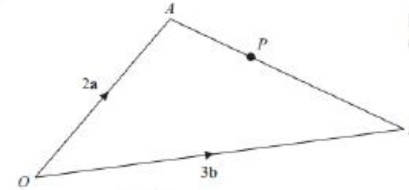


Diagram NOT accurately drawn

(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

P is the point on AB such that $AP : PB = 2 : 3$

(b) Show that $\overrightarrow{OP}$ is parallel to the vector $\mathbf{a} + \mathbf{b}$.

STRETCH

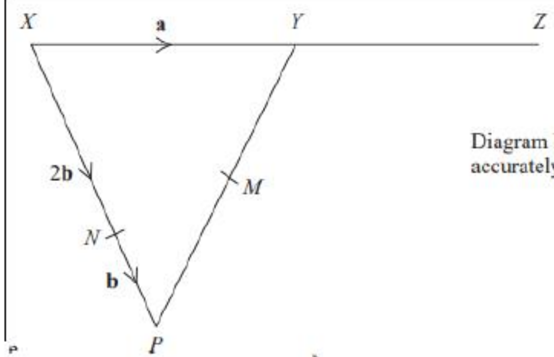


Diagram NOT accurately drawn

(a) Find the vector $\overrightarrow{PX}$, in terms of **a** and **b**.

Y is the midpoint of XZ
M is the midpoint of PY

(b) Show that NMZ is a straight line.

MATHS (Higher) SP - TOPIC 22

Circle Theorems

INTRODUCTION

Circle theorems allow us to solve geometric problems involving circles. There are seven that you need to know.

KEY WORDS

Subtended	An angle between two chords is subtended by the arc between
Inscribed angles	The inscribed angle of the arc that is opposite it.
Intercepted arc	The arc opposite the inscribed arc.
Cyclic quadrilateral	A cyclic quadrilateral is a quadrilateral drawn inside a circle. Every vertex(point) must touch the circumference.

FURTHER LINKS

You can find all circle theorems here:
https://www.bbc.com/bitesize/guides/zcsgdxs/revision/1

Hegarty maths clips: 817-820, 594-606

Corbett maths videos: 64-65

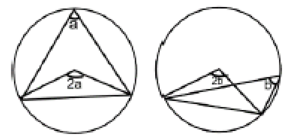
EXAM TIPS

Tip: When calculating angles using a circle theorem, **always** state which theorem applies.

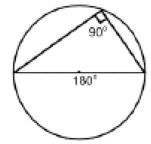
Tip: Make sure you are familiar with the **parts of the circle**: tangent, chord, arc, radius, diameter, circumference, segment, and sector.

KEY FACTS TO MEMORISE

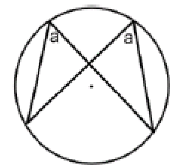
1. The angle subtended by an arc at the centre is twice the angle subtended at the circumference.



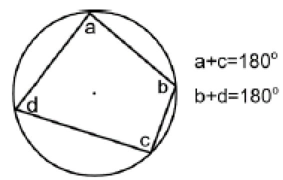
1a. The angle at the circumference in a semicircle is a right angle.



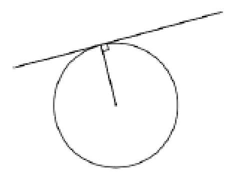
2. The angles at the circumference subtended by the same arc are equal.



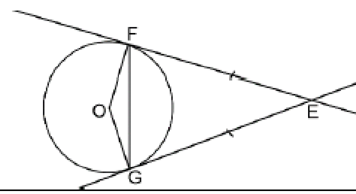
3. The opposite angles in a cyclic quadrilateral add up to 180°.



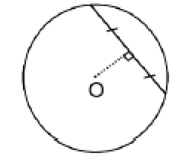
4a. The angle between a tangent and a radius is 90°.



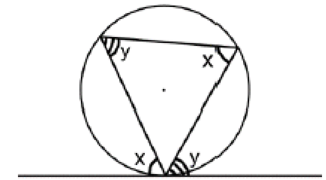
5. Tangents that meet at the same point (outside the circle) are equal in length.



6. The perpendicular from the centre of a circle to a chord bisects the chord

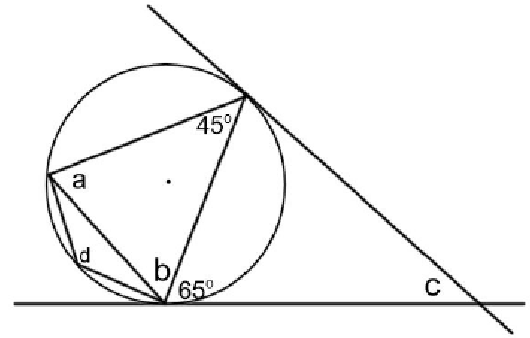


7. The angle between a tangent and a chord is equal to the angle in the alternate segment.



STRETCH-Exam Question

Calculate the angles a, b, c and d.



MATHS (Higher) SP - TOPIC 23

Ratio

INTRODUCTION

A ratio is a relationship between two or more measures. It is expressed as x : y and is directly linked to proportional reasoning.

In this Starting point you will learn how to :

- Simplify a ratio
- Share in a given ratio
- Find one part of a ratio when you have the other
- Express a ratio in the form 1:n or n:1
- Link ratios and fractions

KEY WORDS

Quotient	Result of a division.
Ratio	The quantitative relationship between two amounts.
Equivalent	This when all quantities in a ratio can be divided or multiplied to give an equivalent ratio.
Direct proportion	Direct proportion occurs when one value increases and the other increases.
Inverse proportion	Inverse proportion occurs when one value increases and the other decreases.

FURTHER LINKS

Hegarty clips: 328-337

Corbettmaths video : 269-271

EXAM TIPS:

Simplest form

When leaving a ratio in its simplest form, make sure you only have integers(not decimal numbers).

When given 2 ratios e.g. b:g is 3:2 and g:d is 1:4,
Make the common variable the same number
i.e. 3:2 1:4
3:**2** **2**:8
The ratio b:g:d is 3:2:8.
You can then compare, share and problem solve.

EXAMPLES

Simplifying

Divide all parts by their Highest Common Factor

Sharing

Add the ratio parts. Divide the amount you want to share, by the sum of the parts. Multiply all parts by this quotient.
E.g. Share £130 in the ratio 2:3:5
2+3+5=10 ,
£130÷10=£13 (check your answer by adding 2:3:5 x13
your parts 26+39+65=130)
£26:£39:£65

Sharing-find one part

Sue and Dave share some money in the ratio 1:3. Dave got £21, how much did Sue get?
S : D
2 : 3
x : 21
21÷3=7 , 2x7=£14
Sue got £14.

Difference

Monet and Nina share some money in the ratio 3 : 7. Nina got £20 more than Monet. How much did they share?
The difference in their parts is 4 (7-3=4)
One part will therefore be £5 (£20÷4=£5)
Ten parts (3+7) will then be £50 (10x£5=£50)

Exam Questions

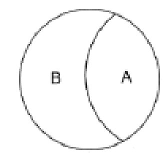
Ratio and problem solving

Problem 1

The angles in a triangle are in the ratio 1:2:9.Find the size of the largest angle.

Problem 2

A : B
2 : 3
The ratio of the area of the regions A and B is 2:3. The radius of the circle is 1.5cm Find the area of A.



Stretch

Problem 3

The ratio of the number of boys to girls at a party is 3:4. Six boys leave the party. The ratio is now 5:8. Work out the number of girls.

MATHS (Higher) SP - TOPIC 24

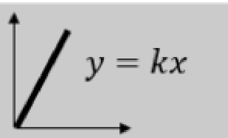
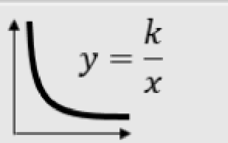
Direct and Inverse Proportion

INTRODUCTION

There is a direct proportion between two values when one is a multiple of the other. Inverse proportion occurs when one value decreases and the other increase. You will need to be able to form equations from a proportional relationship and use this equation to answer questions

KEY WORDS

Proportion	Two variables are proportional if there is always a constant ratio between them
Constant of Proportionality	The constant ratio between the variables, often denoted by k
Ratio	The quantitative relationship between two amounts

	When one variable increases as another increases their proportionality is referred to as **direct**.
	When one variable decreases as another increases their proportionality is referred to as **inverse**.

FURTHER LINKS

Hegartymaths 339 - 348

Corbett Maths 254 - 25

EXAM TIPS:

1) Establish the relationship (direct or inverse) and write the equation
2) Substitute in the specified values from the question to find the constant of proportionality
3) Write out the equation (including the constant)
4) Apply the equation to solve the given problem

KEY FACTS TO MEMORISE

If you have a direct proportion question your equation should begin $y \propto x$, which becomes $y = kx$

If you have an inverse proportion question your equation should begin $y \propto \frac{1}{x}$, which becomes $y = \frac{k}{x}$

Example

A ball is dropped from a tower. After t seconds the ball has fallen a distance of x metres.

x is **directly proportional** to t^2

When t=2, x=19.6

Find an equation connecting x and t

$x \propto t^2$ $\qquad x = kt^2$

$19.6 = k \times 2^2$

$k = \frac{19.6}{4} = 4.9$ $\qquad x = 4.9\,t^2$

Find the value of x when t = 3

$x = 4.9\,t^2$

$x = 4.9 \times 3^2$ $\qquad x = 44.1$m

Find how long the ball takes to fall 10m

$x = 4.9\,t^2$

$10 = 4.9\,t^2$

$t^2 = \frac{10}{4.9}$ $\qquad t = \sqrt{\frac{10}{4.9}}$ $\qquad t = 1.43$ seconds

Example

The number of days ,D, to complete research is **inversely proportional** to the number of researchers, R, who are working. It takes 125 days to complete if 16 people work on it.

Find an equation connecting D and R

$D \propto \frac{1}{R}$ $\qquad D = \frac{k}{R}$

$125 = \frac{k}{16}$ $\qquad k = 2000$

$D = \frac{2000}{R}$

Exam Questions

1) R is directly proportional to S.

When R = 9, S = 1.5.

(a) Find an equation for R in terms of S.

(b) Find R when S = 8

(c) Find S when R = 15

2) y is inversely proportional to x^2

Given that y = 2.5 when x = 24.

(a) Find an equation for y in terms of x.

(b) Find the value of y when x = 20.

Stretch It

a is directly proportional to $\sqrt{c}$.

w is inversely proportional to a^3.

When c = 49, a = 35

When a = 2, w = 16.

Find the value of w when c = 4.

edexcel

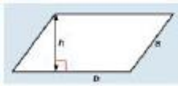
Edexcel GCSE (9-1) Maths: need-to-know formulae

www.edexcel.com/gcsemathsformulae

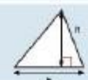
Areas

Rectangle = $l \times w$

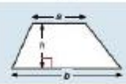
Parallelogram = $b \times h$



Triangle = $\frac{1}{2} b \times h$

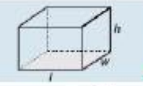


Trapezium = $\frac{1}{2}(a + b)h$

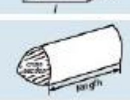


Volumes

Cuboid = $l \times w \times h$

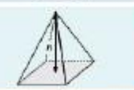


Prism = area of cross section × length



Cylinder = $\pi r^2 h$

Volume of pyramid = $\frac{1}{3}$ × area of base × h

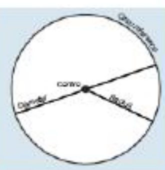


Circles

Circumference = π × diameter, $C = \pi d$

Circumference = $2 \times \pi$ × radius, $C = 2\pi r$

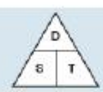
Area of a circle = π x radius squared $A = \pi r^2$



Compound measures

Speed

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$



Density

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

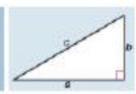
Pressure

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Pythagoras

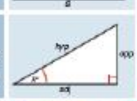
Pythagoras' Theorem

For a right-angled triangle, $a^2 + b^2 = c^2$



Trigonometric ratios (*new to F*)

$\sin x° = \frac{\text{opp}}{\text{hyp}}$, $\cos x° = \frac{\text{adj}}{\text{hyp}}$, $\tan x° = \frac{\text{opp}}{\text{adj}}$

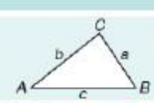


Trigonometric formulae

Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$



Quadratic equations

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2-4ac)}}{2a}$

Foundation tier formulae

Higher tier formulae

Notes page

Y11 GCSE Exam Dates

Y11 Mock(s): ____________________

Y11 PPE(s): ____________________

Final GCSE(s): ____________________

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Success Programme Sessions:

Revision Guide (if applicable):

Notes